Solving practical decision problems under severe uncertainty

Some applications of imprecise probability in the environmental and engineering sciences

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Outline

Research Interest

Imprecise Probability: What & Why?

Case Study: Robust Modelling of Wind Power to Quantify Long Term Energy Security

- Power System Adequacy
- Wind Power Modelling
- Conventional Generation Modelling
- Demand Modelling
- Simulation Results

Case Study: Land Use Modelling

- Land Use Model
- Data
- Robust Bayesian Analysis

Conclusion

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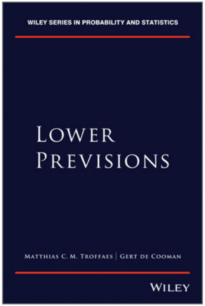
Research Interests

- quantifying uncertainty using imprecise probability
  lower/upper previsions, robust Bayesian methods
- mathematical methods for imprecise probability
  non-linear functionals, convex analysis
- decision making
  methods, algorithms, graphical models, dynamic programming, consistency
- applications of any of the above

[9]



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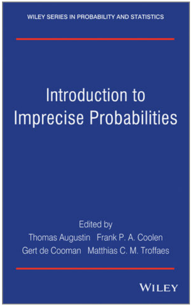

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Research Interests: Uncertainty

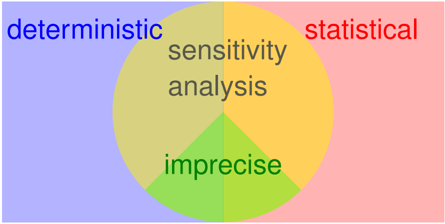
how to provide reassurance
that your mathematical models apply to the real world?

Statistics

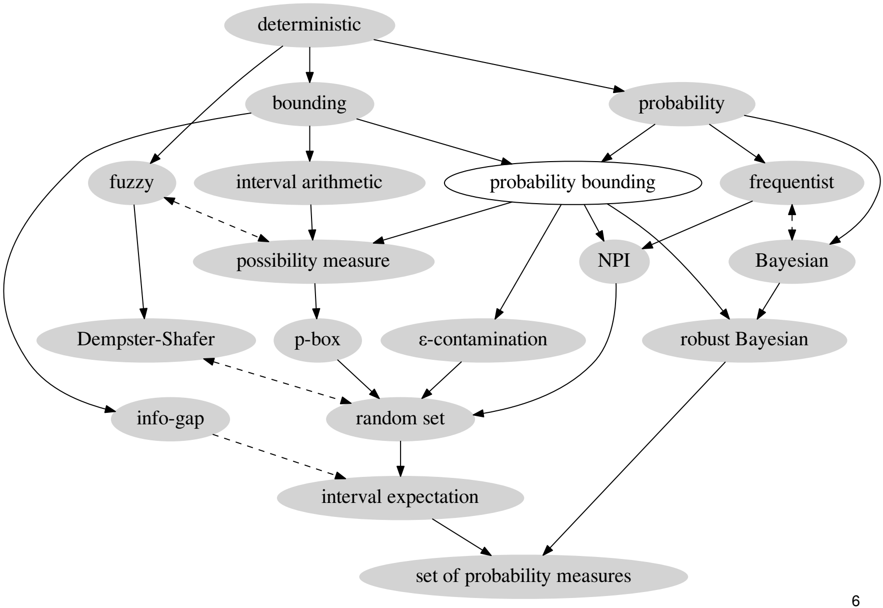
provides tools for checking this systematically
aim: reasonable reassurance—guarantees are impossible

Imprecise Probability

for the bits that are really hard to quantify
partial expert opinion, sparse data



The Uncertainty Zoo (term coined by John Aldridge)



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Requirements for an Uncertainty Model

Operational

How can uncertainty be reliably

- measured?
- communicated?

Inference

How can we use our theory of uncertainty for

- statistical reasoning?
- decision making?

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Probability: Interpretations

Interpretation: Trivial Cases

$P(A) = 0 \iff A$ is false
$P(A) = 1 \iff A$ is true

what about values between 0 and 1, such as $P(A) = 0.2$?

Interpretation: General Case

- it's a relative frequency ('objective probability', 'chance')
- it's a betting rate ('subjective probability')
- it's something else

Key Problem

- getting probabilities needs plenty of data (or plenty of elicitation)

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Aim

do statistics with partial elicitation and/or sparse data

many answers:

- strong model assumptions
- likelihood (frequentists)
- prior + likelihood (Bayesians/Laplacians)
- expectation + covariance (Bayes-linear-ists)
- non-parametric (Wilcoxon, NPI)
- interval probabilities (Booleans)

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Imprecise Probability: Sensitivity Interpretation

Definition

A credal set $\mathcal{M}$
is a set of probability measures.

Sensitivity Interpretation of $\underline{P}$

One of the probability measures P in the credal set $\mathcal{M}$ is correct,
but we do not know which one.

For instance, we may be able to exclude some distributions based on the data, but we do not have enough information to exclude all but one.

crucial: no distribution over $\mathcal{M}$ assumed!
(why not?)

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Imprecise Probability: Summary of Main Issues

- How to do statistics with partial elicitation and sparse data?
- Use of lower and upper probability appears, at least naively, to be a simple way of dealing with sparse data in principle.
- How do you actually get the lower and upper bounds 'just from' data?
- *How can we use these models in decision making?*

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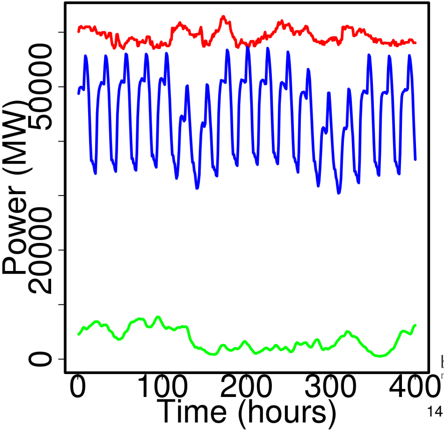
Power System Adequacy

What & Why?

- energy shortage = total generation < total demand
- statistical quantification of possible future energy shortages?
- useful for long term power system planning



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Power System Adequacy

How

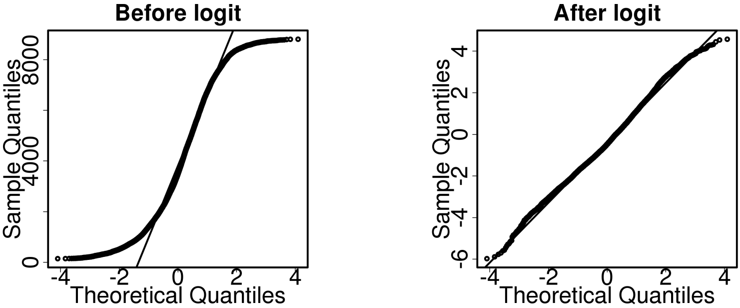
- Build models for different types of generation capacity and demand.
- Fit parameters to data.
- Simulate future scenarios.
- Summarize via any suitable adequacy risk index
  (total energy not served, frequency and duration of outages, . . .)

Difficulties

- Requires full time-series model of capacity and demand.
- Quantify future capacity (substantial increase in renewables).
- Limited data to validate all model assumptions.
- Utility function for power loss?
- Contribution of storage?

Wind Power Modelling

- Data from "Adjusted Gone Green" scenario supplied by National Grid (representative of UK but not the actual observations)
- UK total wind power data for 7 winters; each winter is 20 weeks.
- Classical approach: ARMA. Issue: marginal not normal. [7, 8] **(Why?)**



$$\mathrm{logit}(x) = \log\left(\frac{t(x)}{1-t(x)}\right) \text{ where } t(x) = \frac{x-\alpha}{\beta-\alpha}. \quad (1)$$

$[\alpha, \beta]$ fitted by trial and error

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Wind Power Modelling

logit(wind power) = winter mean + zero-mean ARMA process

years are not exchangeable!!!

- mean varies across years, may or may not be normal (why?)

y	2005	2006	2007	2008	2009	2010	2011
$\widehat{\text{mean}}$	-0.62	0.22	0.03	-0.56	-0.66	-0.84	-0.17
error	± 0.33	± 0.56	± 0.42	± 0.38	± 0.45	± 0.32	± 0.61

- ARMA parameters vary across years, normality fairly good

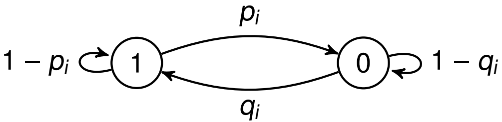
y	$\hat{\alpha}_1(y)$	$\hat{\alpha}_2(y)$	$\hat{\alpha}_3(y)$	$\hat{\alpha}_4(y)$	$\hat{\alpha}_5(y)$	$\hat{\sigma}(y)$
2005	2.54	-2.54	1.48	-0.64	0.16	0.04
2006	2.56	-2.65	1.63	-0.7	0.16	0.06
2007	2.49	-2.45	1.38	-0.55	0.12	0.06
2008	2.41	-2.25	1.17	-0.44	0.09	0.06
2009	2.56	-2.58	1.46	-0.58	0.14	0.04
2010	2.53	-2.51	1.39	-0.54	0.12	0.04
2011	2.22	-1.73	0.68	-0.27	0.09	0.06

we use sensitivity analysis (imprecise probability [12, 9])
to deal with lack of exchangeability

Conventional Generation Modelling

Assumptions

- Each unit W_i follows a 2 state discrete time Markov chain.



- Units are independent.

$$X(t) = \sum_{i=1}^{k} c_i W_i(t). \tag{2}$$

Issues

- Assumptions clearly violated in practice! Not addressed yet.
- Parameter estimation?
  Capacities and availabilities from National Grid.
  Remaining parameters naively fitted from literature [1].
- Simulation of 300 independent Markov chains is very slow.

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Demand Modelling

Methodology

- Complicated!! (daily cycle, weekly cycle, yearly cycle, holidays. . .) [4]
- Simple approach to condition on an actual demand trace: hindcasting
- Adjust for future demand by multiplicative scaling.

'average cold spell peak'

Issues

- How to justify exact scaling values?
- Scaling provides only one handle
  to control mean, variance, and epistemic uncertainty about these.
- Future demand traces may look very different from any observed year.

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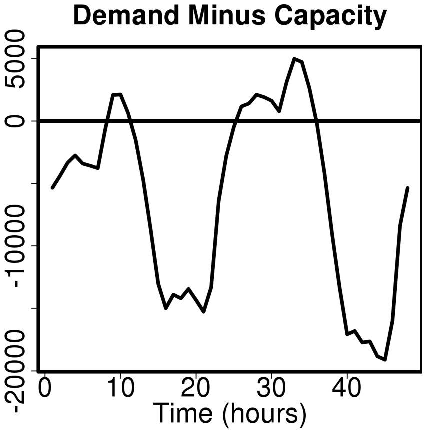
Simulation Results



Two risk indices considered:

- energy not served E = area under curve above horizontal axis
- number of shortfalls N = number of such areas

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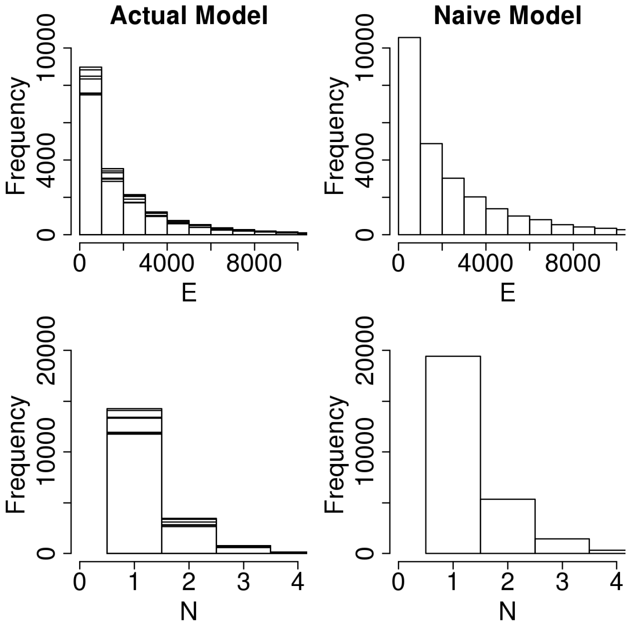
Simulation Results: 100 000 Generated Winter Traces



Simulation Results: Expected Energy Not Served

$$\underline{P}(E) = \min_{y=1}^{7} \frac{1}{n} \sum_{i=1}^{n} \sum_{t=1}^{3360} \max\left\{0, d(t) - c_i(t) - w_{yi}(t)\right\} = 299.63 \pm 9.24 \tag{3}$$

$$\overline{P}(E) = \max_{y=1}^{7} \frac{1}{n} \sum_{i=1}^{n} \sum_{t=1}^{3360} \max\left\{0, d(t) - c_i(t) - w_{yi}(t)\right\} = 389.91 \pm 9.24 \tag{4}$$

$$\underline{P}(E = 0) = 0.813 \pm 0.002 \tag{5}$$

$$\overline{P}(E = 0) = 0.848 \pm 0.002 \tag{6}$$

For comparison, the naive model has:

$$P(E) = 808.416 \pm 16.501 \qquad P(E = 0) = 0.733 \pm 0.003 \tag{7}$$

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Discussion

- Power modelling presents a wonderful statistical challenge.
- Statistical assumptions are important.

  Naive model overestimates risk by a factor of more than 2.
- Imprecision can handle, to some extent, loss of exchangeability.

  No theory yet to back this up.
- Do engineers believe too much in their models?
- Utility model for energy shortages? Whose risk?

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Land Use Modelling & Crop Rotation

joint work with Lewis Paton & Andy Hart & Nigel Boatman & Mohammud Hussein [10, 6]

- aim: model and predict agricultural land use
- why? food security, landscape, environmental impact

Problems

an abundance of uncertain factors influencing crop choices

Factors Influencing Crop Choice in a Particular Field

- soil type
- previous years' crops
- intensity & time of rainfall
- temperature
- crop price
- fertilizer price
- farmer's attitude towards risk

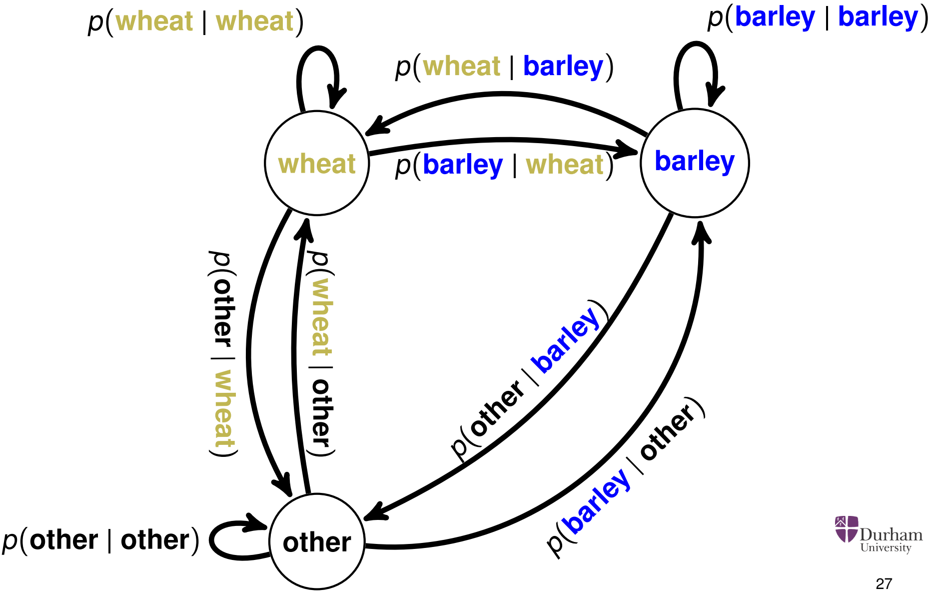
The Model

- crop sequences typically follow set patterns

	year 1	year 2	year 3	year 4
field 1	**wheat**	**fallow**	**wheat**	**beans**
field 2	**barley**	**barley**	**sugar beet**	**wheat**
field 3	**grass**	**grass**	**wheat**	**grass**

- patterns not entirey deterministic: we use a Markov chain

Land Use Model (Simplified)



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Data

- historical crop data
- historical rainfall data (needs spatial interpolation)
- historical fertiliser price data
- soil type map
- expert information on historic crop profit predicitions
- expert information on yield level per crop and soil type
- predictions of future price and climate scenarios

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Robust Bayesian Analysis

- key idea: probabilities are a logistic function of a linear combination of the continuous factors (climate and price) influencing crop choices

$$p(\textbf{wheat} \mid \textbf{barley}) = \text{function of } \beta_0 + \beta_1 \times \text{price} + \beta_2 \times \text{climate}$$

- aim of statistical inference: identify β_0, β_1, and β_2 from data
- only limited prior information about β's:
  sensitivity analysis over a near-vacuous class of priors
  MAP estimation because Monte Carlo simulation is too expensive

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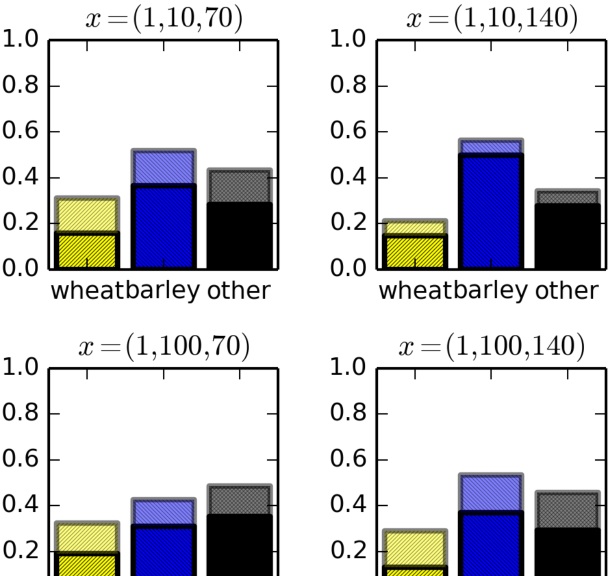
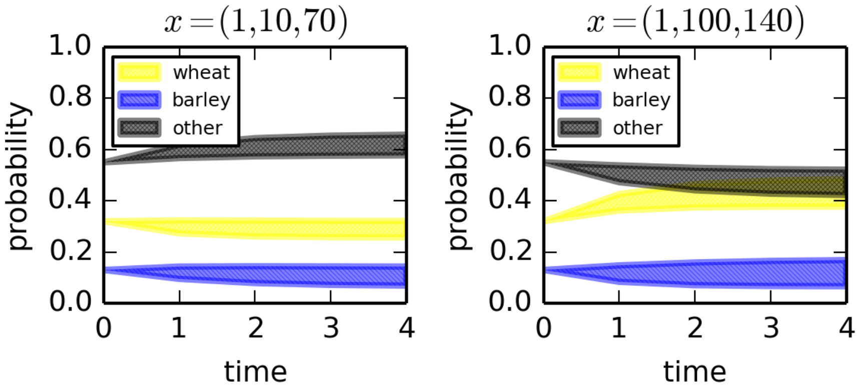
Robust Bayesian Analysis:
Transition Probabilities from **barley**



Robust Bayesian Analysis:
Future Crops under Different Scenarios



Robust Bayesian Analysis: Policy Decision Support

- interested in stimulating increase in legumes
- utility function:

$$U(a,b) = a - \kappa b$$

a = fraction of legumes across all farms;
function of b and model parameters β
b = subsidy level
κ = weight constant

- maximize expected utility by considering all β^* MAP estimates:

$$\left\{\arg\max_b U(a(b,\beta^*),b) : \beta^* \in B^*\right\}$$

range of optimal policy recommendations
in most cases, actually a unique policy identified

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Conclusion

Is imprecise probability useful?

Should we combine bounding and probability
when data is sparse and expert information is limited?

- Increased ‘confidence’ in analysis.
- Harder to communicate uncertainty?
- Harder to compute.
- Relatively immature theory: fewer off the shelf results.

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Thank you!

Selected Publications I

[1] A. B. Attya, Y. G. Hegazy, and M. A. Moustafa.
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[2] Thomas Augustin, Frank P. A. Coolen, Gert De Cooman, and Matthias C. M. Troffaes, editors.
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[3] R. Billinton, Yi Gao, and R. Karki.
Composite system adequacy assessment incorporating large-scale wind energy conversion systems considering wind speed correlation.
IEEE Transactions on Power Systems, 24(3):1375–1382, August 2009.
doi:10.1109/TPWRS.2009.2023263.

[4] Ching-Lai Hor, S.J. Watson, and S. Majithia.
Analyzing the impact of weather variables on monthly electricity demand.
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doi:10.1109/TPWRS.2005.857397.

[5] Rajesh Karki, Po Hu, and Roy Billinton.
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IEEE Transactions on Energy Conversion, 21(2):533–540, June 2006.

[6] Lewis Paton, Matthias C. M. Troffaes, Nigel Boatman, Mohamud Hussein, and Andy Hart.
Multinomial logistic regression on Markov chains for crop rotation modelling.
In Anne Laurent, Oliver Strauss, Bernadette Bouchon-Meunier, and Ronald R. Yager, editors, *Proceedings of the 15th International Conference IPMU 2014 (Information Processing and Management of Uncertainty in Knowledge-Based Systems, 15–19 July 2014, Montpellier, France)*, volume 444 of *Communications in Computer and Information Science*, pages 476–485. Springer, 2014.
doi:10.1007/978-3-319-08852-5_49.

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[7] P. Pinson.
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Journal of the Royal Statistical Society: Series C (Applied Statistics), 61(4):555–576, 2012.
doi:10.1111/j.1467-9876.2011.01026.x.

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[10] Matthias C. M. Troffaes and Lewis Paton.
Logistic regression on Markov chains for crop rotation modelling.
In F. Cozman, T. Denœux, S. Destercke, and T. Seidenfeld, editors, *ISIPTA'13: Proceedings of the Eighth International Symposium on Imprecise Probability: Theories and Applications*, pages 329–336, Compiègne, France, July 2013. SIPTA.
URL: http://www.sipta.org/isipta13/index.php?id=paper&paper=033.html.

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[12] Peter Walley.
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