Deciding under ignorance : in search of meaningful extensions of the Hurwicz criterion to decision trees

Didier Dubois Hélène Fargier
Romain Guillaume Caroline Thierry

IRIT, Université Toulouse, France

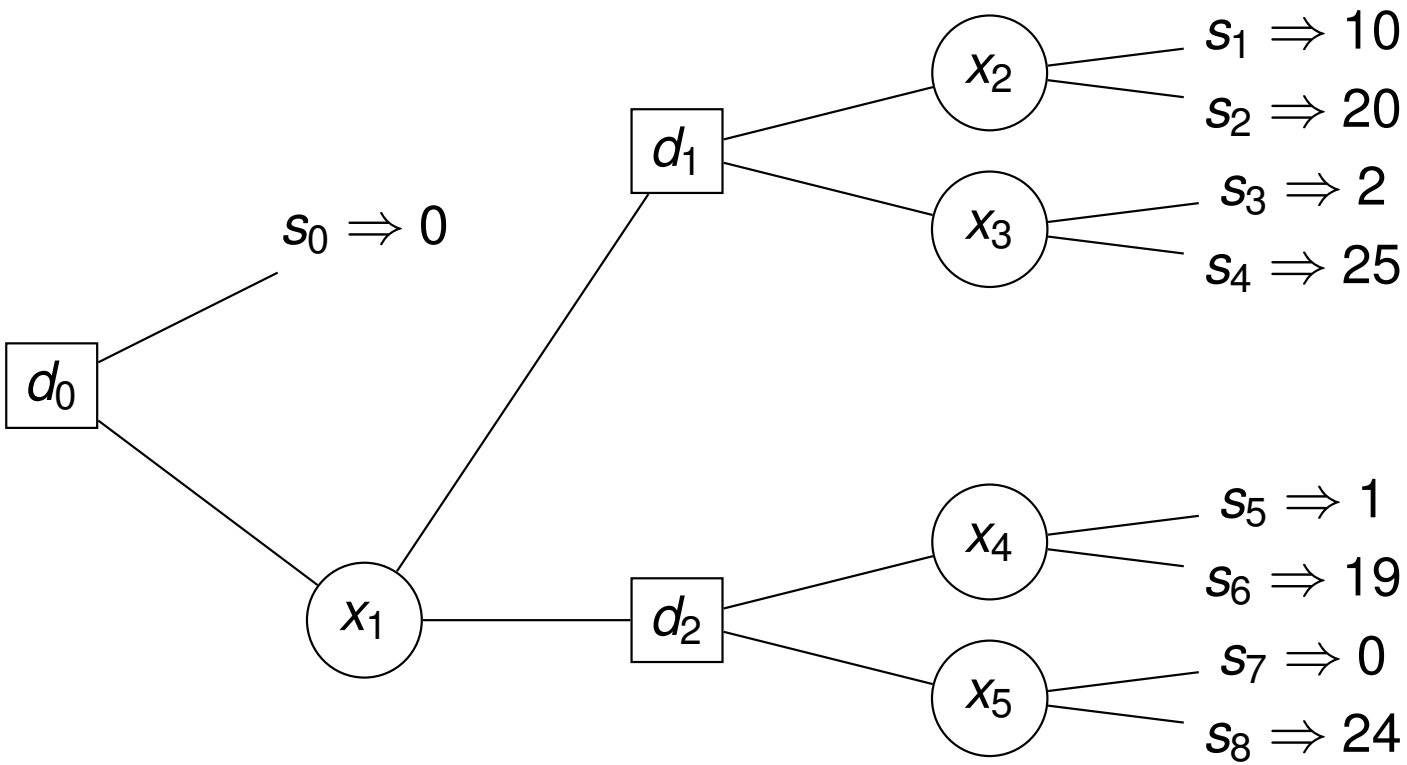
26 mai 2015

Contents

Decision Tree and Requested properties for strategies

- A decision tree is a tree whose nodes are decision nodes (where DM chooses) followed by chance nodes (where nature chooses).
- A set of decisions one per decision node is called a strategy.
- Three principles are supposed to be respected :
  - Dynamic consistency
  - Consequentialism
  - Global optimality criterion based on tree reduction for each strategy

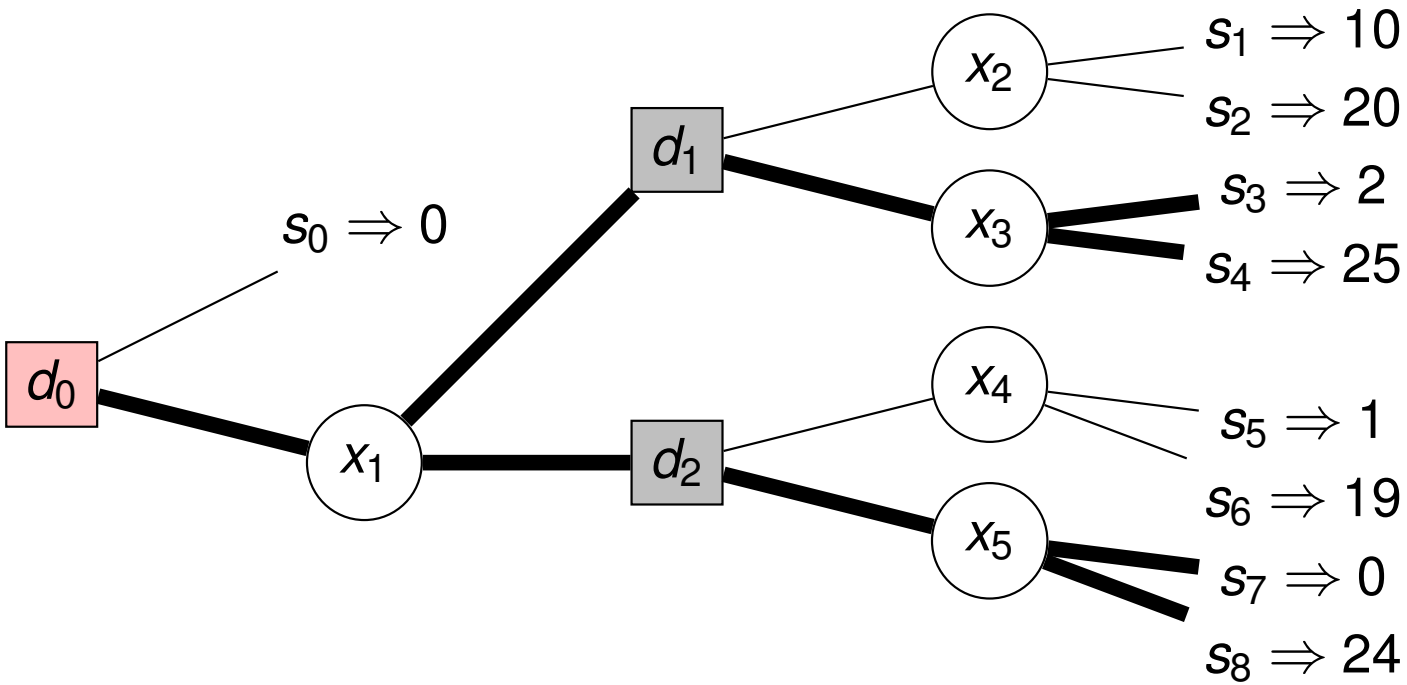
Decision Tree



Dynamic Consistency

Definition (Dynamic Consistency)

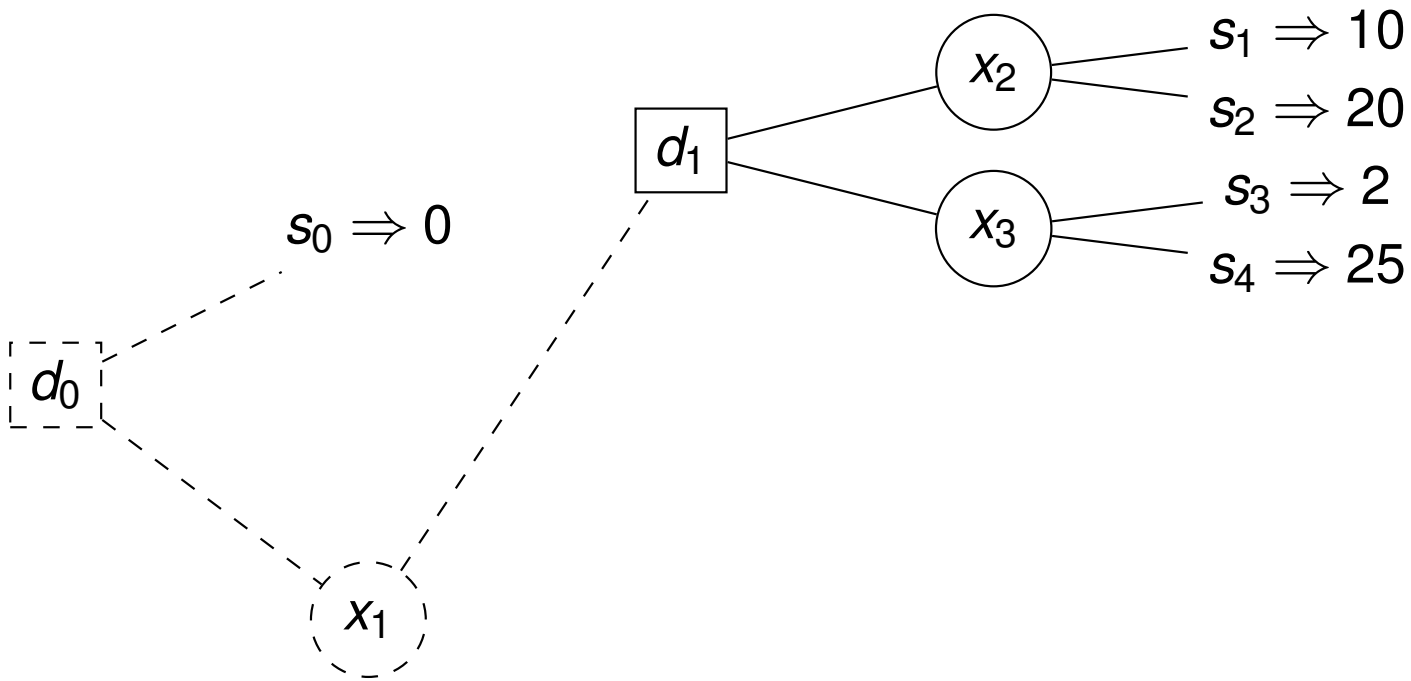
When reaching a decision node by following an optimal strategy, the best decision at this node is the one that had been considered so when computing this strategy, i.e. prior to applying it.



Consequentialism

Definition (Consequentialism)

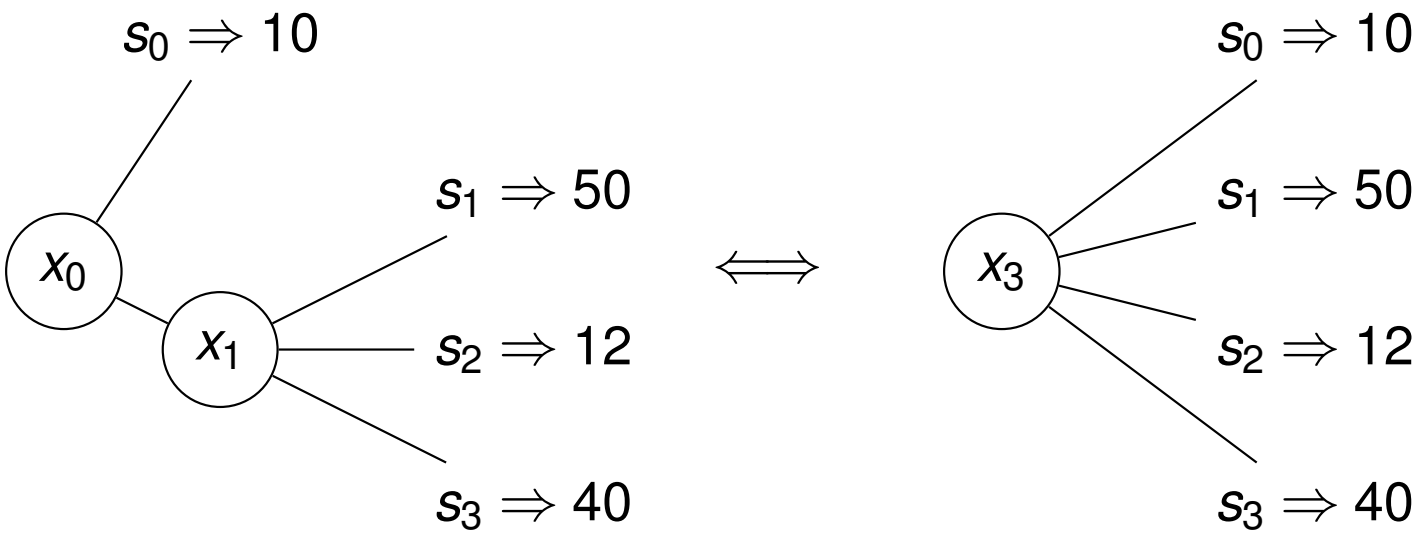
the best decision at each step of the decision tree only depends on potential consequences at this point.



Tree Reduction

Definition (Tree reduction)

The evaluation of strategies is done on the corresponding reduced uncertainty tree.

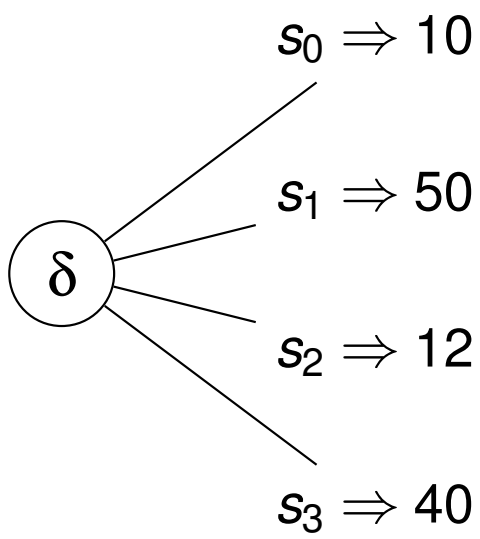


Hurwicz criterion

An old criterion under total uncertainty (no probabilities available).

$$H(\delta) = \alpha \times \min_{s \in E_\delta} u(s) + (1 - \alpha) \times \max_{s \in E_\delta} u(s).$$

A compromise between optimistic and pessimistic criteria.
α is the degree of pessimism of the DM.

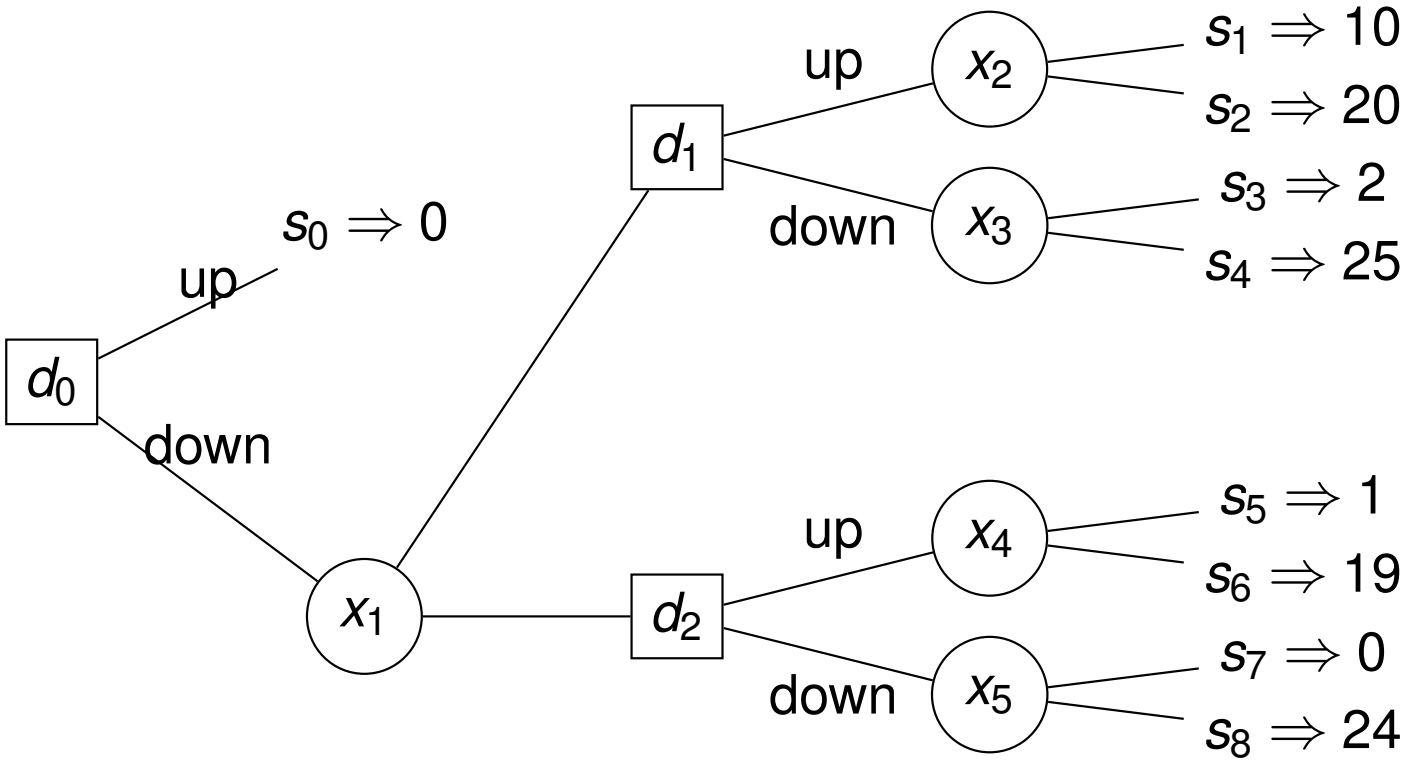


$$\Rightarrow H(\delta) = \alpha \times 10 + (1 - \alpha) \times 50$$

Resolute Choice

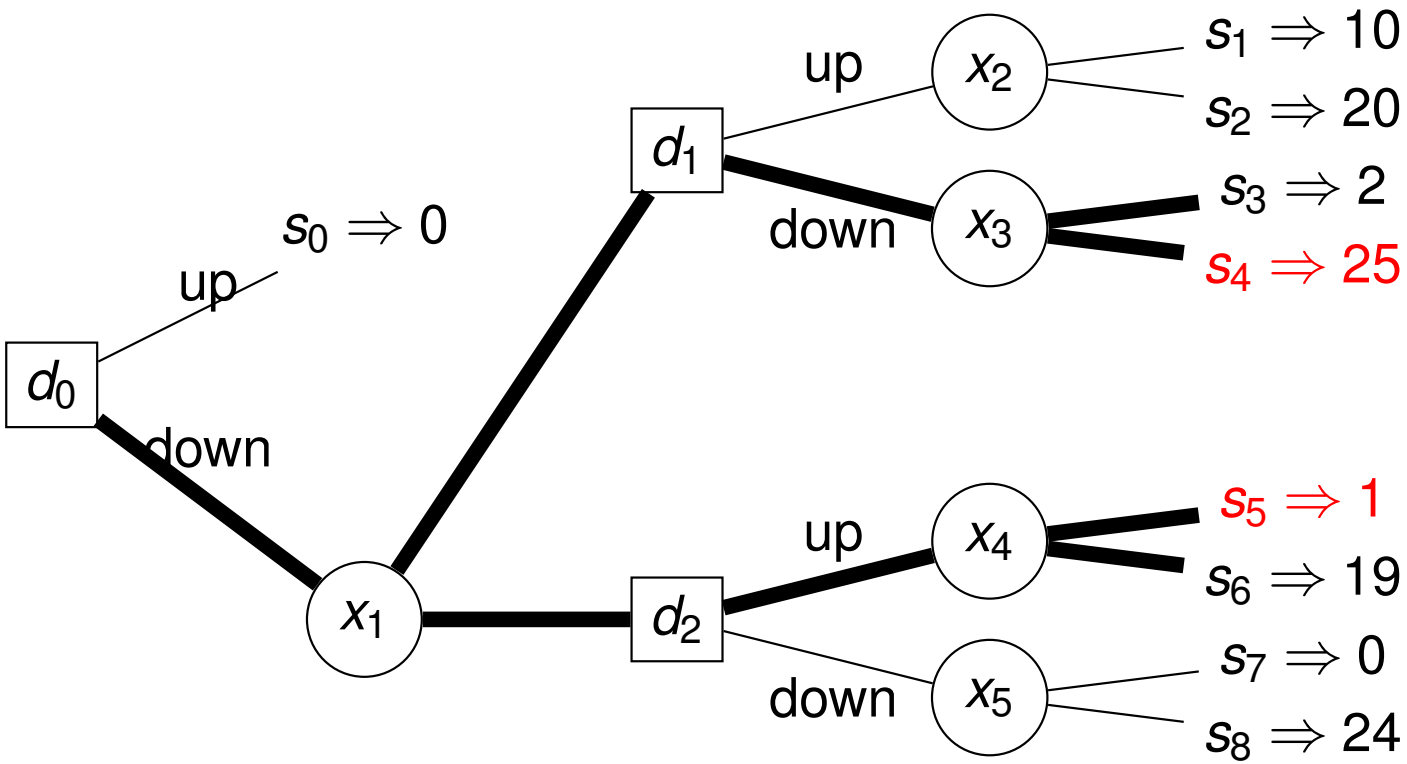
Resolute Choice

Apply lottery reduction to the compound lottery then evaluate the strategy using the Hurwicz criterion



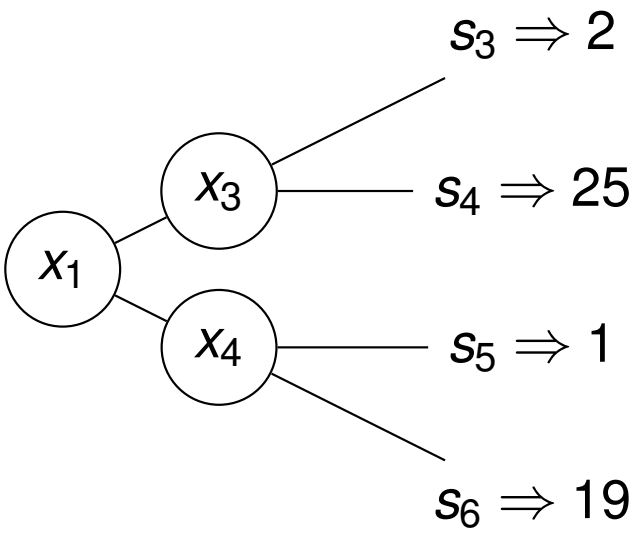
Resolute Choice

Apply lottery reduction to the compound lottery then evaluate the strategy using the Hurwicz criterion



Resolute Choice

This is the best solution regardless of the value of α.



$$\Longleftrightarrow \quad H = \alpha + 25(1 - \alpha)$$

The paradox of resolute choice

Optimal solution of Resolute Choice

$d_0 = down, d_1 = down, d_2 = up$

The decision maker reaches decision node d_1

- $H(\mathbf{d_0} = \mathbf{down}, up) = 10\alpha + 20(1-\alpha)$
- $H(\mathbf{d_0} = \mathbf{down}, down) = 2\alpha + 25(1-\alpha)$

Optimal decision for decision node d_1

If the decision-maker is pessimistic enough ($\alpha > \frac{5}{13}$) the best decision d_1 is "up"

Motivation for alternative approach

optimal solution of Resolute Choice

$d_0 = down, d_1 = down, d_2 = up$

The decision maker reaches decision node d_2

- $H((\mathbf{d_0} = \mathbf{up}, d_2 = up)) = \alpha + 19(1 - \alpha)$
- $H(\mathbf{d_0} = \mathbf{up}, d_2 = down)) = 24(1 - \alpha)$

Optimal decision for decision node d_2

If the decision-maker is enough optimistic $\alpha < \frac{5}{6}$ the best decision d_2 is "down"

Lesson drawn

- Dynamic consistency, consequentialism and tree reduction are conflicting in the presence of total uncertainty
- Expected utility criterion satisfies the three principles : dynamic programming computes optimal strategies
- Non-expected utility criteria : the tree principles are in conflict. Usually consequentialism is dropped and the optimal strategy is computed by other more complex methods

Our thesis : We should keep dynamic consistency and consequentialism because the structure of the tree contains information used by the decision maker.

Veto-process under pure uncertainty

The idea that tree reduction should be dropped was investigated by Jaffray (1999) using "egos"
An ego (or self) is a representation of the decision-maker at various time points

Veto-process

- Assigning a different ego to each decision node
- A ego can not choose a strategy containing one eliminated by future egos
- A ego chooses a sub-strategy by
  1. applying lottery reduction to the compound lottery from its standpoint
  2. then evaluating his sub-strategies using the Hurwicz criterion
  3. eliminating non-optimal sub-strategies

Illustration of Veto-process under pure uncertainty

The approach proceeds backwards from the leaves.

Optimal decision for future egos in nodes d_1 and d_2 with $\alpha \in]\frac{5}{13}, \frac{5}{6}[$

- the best decision d_1 is "up"
- the best decision d_2 is "down"

Optimal strategy

$d_0 = down$

- if $x_1 = up$ then $d_1 = up$
- if $x_1 = down$ then $d_2 = down$

Discussion on Veto-process under pure uncertainty

The Veto of future egos enforces Consequentialism

Egos are like independent players, but are they ?

- In sequential decision under uncertainty all players are dependent
- Only the final player gets the final reward

Ego-dependent process under pure uncertainty

Ego-dependent process under pure uncertainty

- The preference degrees of one ego is a function of the satisfaction degree of its future egos
- The current decision must put the future egos in the best position to be satisfied

- Apply the criterion to optimal substrategies from the leaves back
- Use the rating of the best substrategy as a certainty-equivalent reward for the next computation step
- Apply the criterion in a nested way, so the result does depend on the tree structure.

Illustration of Ego-dependent process under pure uncertainty

Back to our example with $\alpha = \frac{6}{13}$

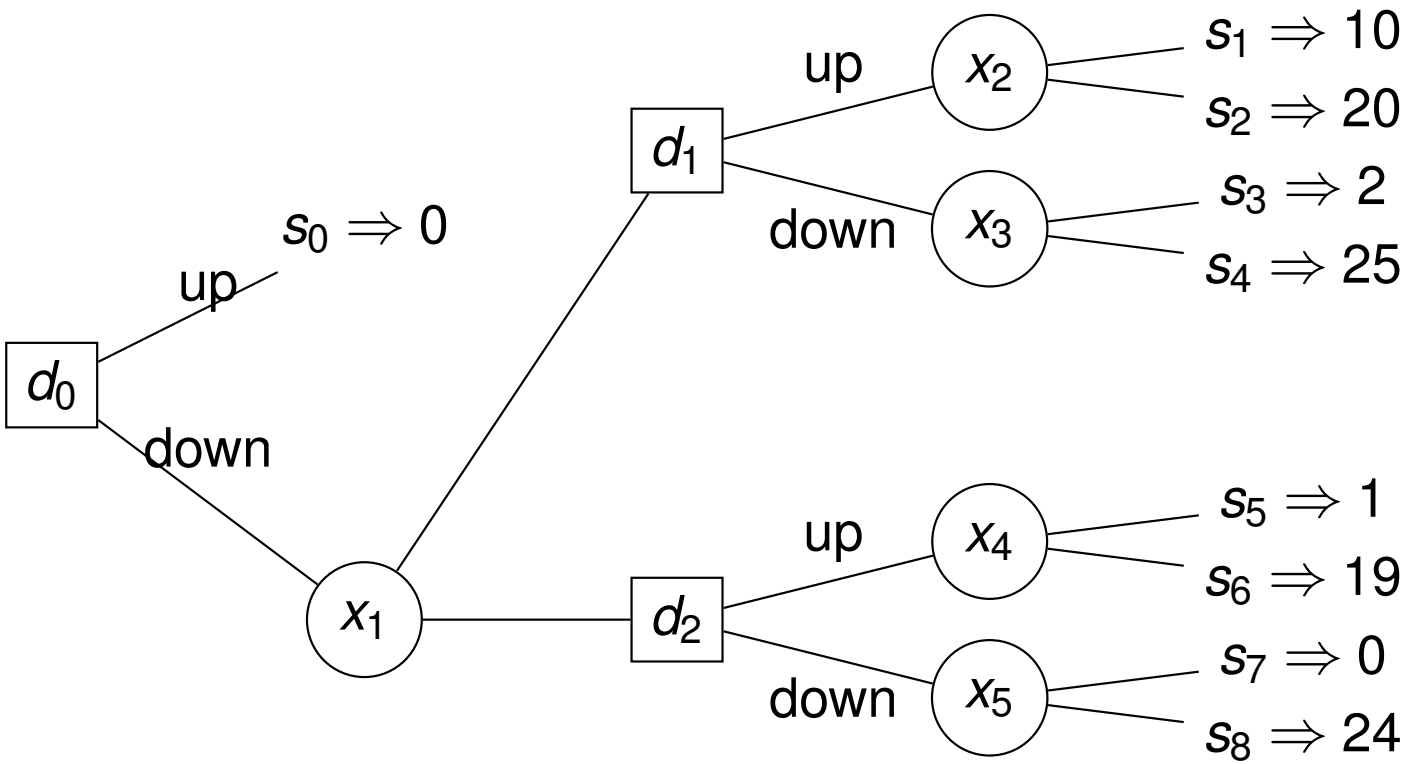


Illustration of Ego-dependent process under pure uncertainty

Back to our example with $\alpha = \frac{6}{13}$

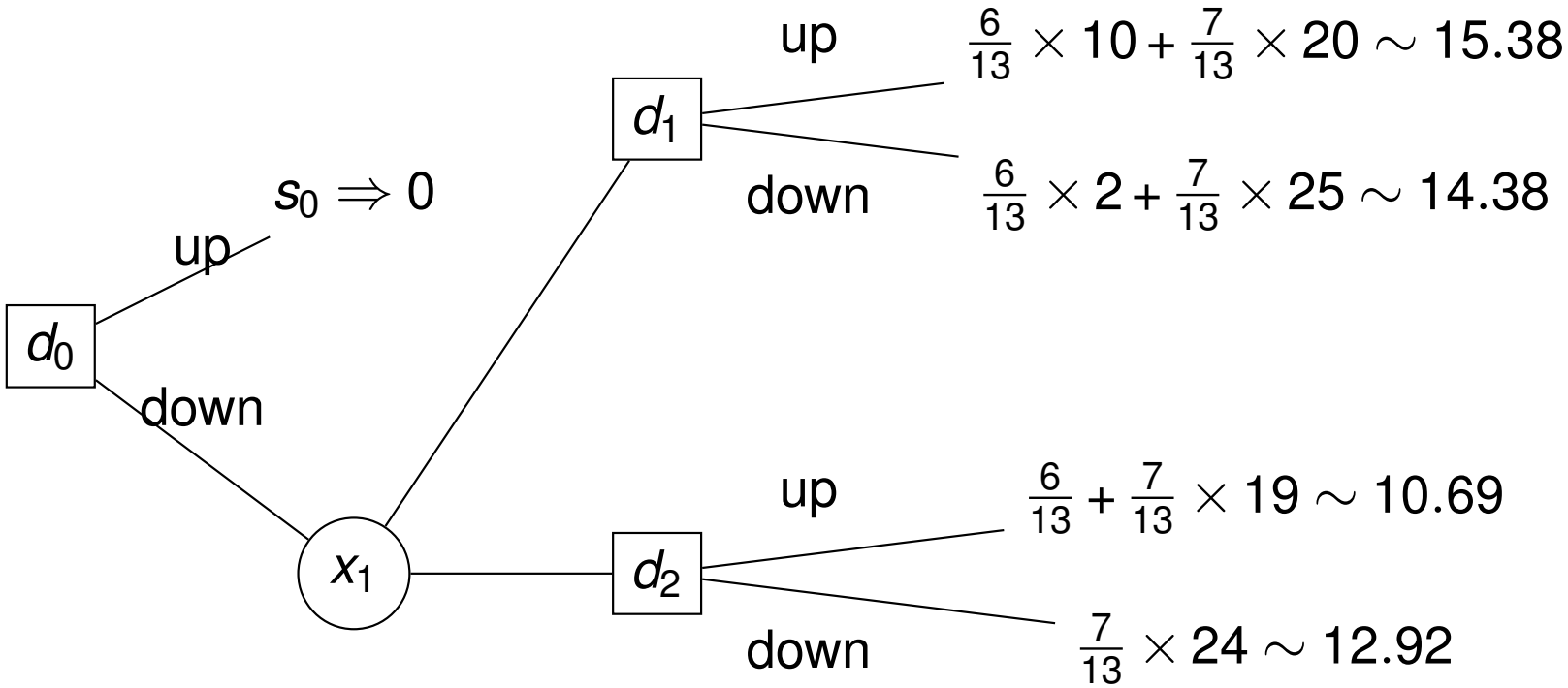
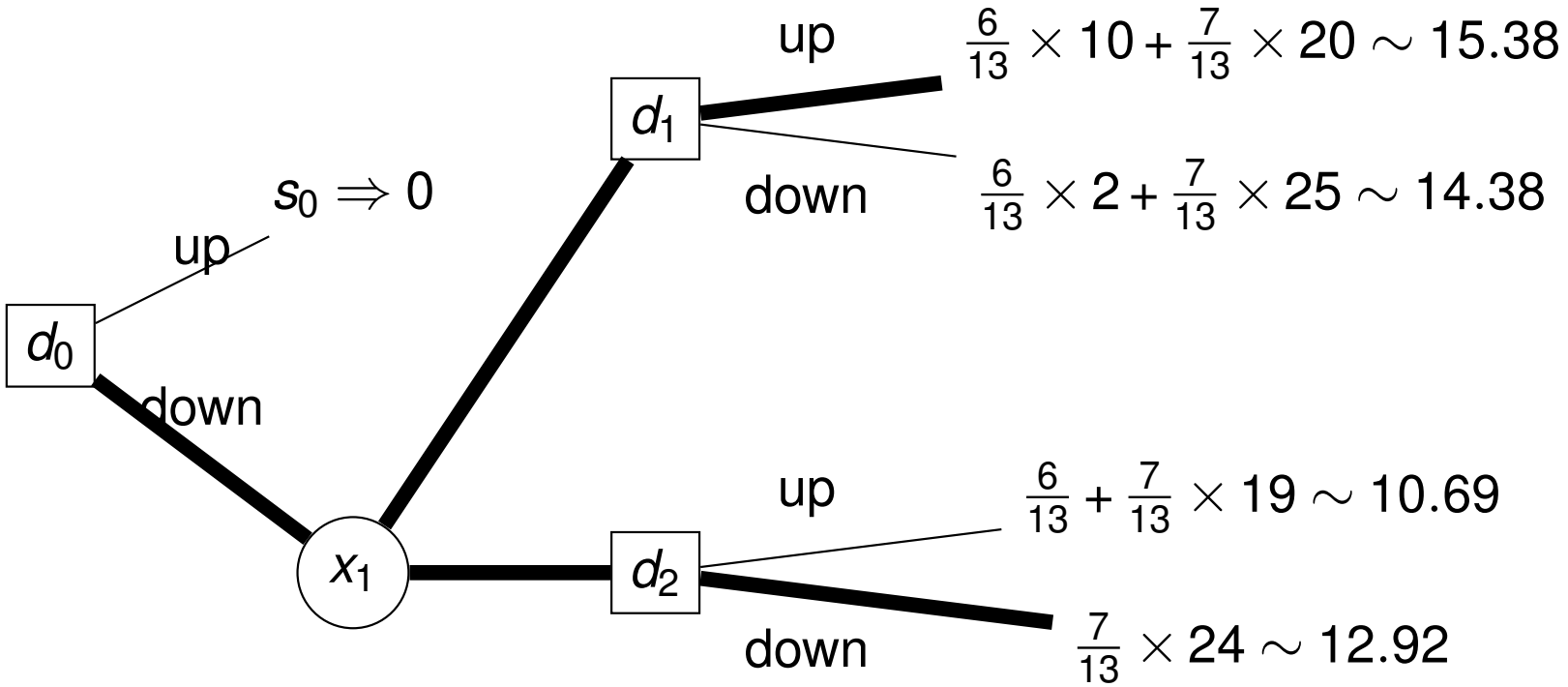


Illustration of Ego-dependent process under pure uncertainty

Back to our example with $\alpha = \frac{6}{13}$



Properties of Ego-dependent process

Verified

- Dynamic Consistency : Any substrategy of optimal stategy is optimal with respect to the corresponding decision subtree
- Consequentialism : The optimal strategy can be computed by means of dynamic programming from leaves to the root of the decision tree

Properties of Ego-dependent process

Failed

Tree Reduction : The structure of the decision tree affects the choices of the decision-maker in the presence of incomplete information, because its state of information about the future changes from one node to another.

Tree reduction is more technical than intuitive : it eliminates the information on the tree structure

Due to incomplete information, some aspects of future decisions are drowned in uncertainty, and reappear at future points when past uncertainty has vanished

Wrapping up

Conclusions

- Resolute Choice is not always psychologically acceptable
- Lottery reduction eliminates causal information embedded in decision trees (Shafer)
- The Veto process enforces Consequentialism
- The ego-dependent process takes into account the structure of decision trees

Shafer 's The Art of Causal Conjecture (MIT press, 1996) reminds that at the origin, probability distributions were described by trees (Huygens, 1976), not probability measures, and the latter loses the causal content of the tree.

Future research

Perspectives

- From static criteria, define global decision criteria that account for the decision tree structure
- Study the characteristic properties of solutions returned by Ego-dependence process
- Study other utility functionals under the same Ego-dependent process